Cryptography

Contents

1. Basic Terminology

Basic Terminology

- **Plaintext:** The original message
- **Ciphertext:** the crypted message
- **Encryption**: the process of converting plaintext into ciphertext

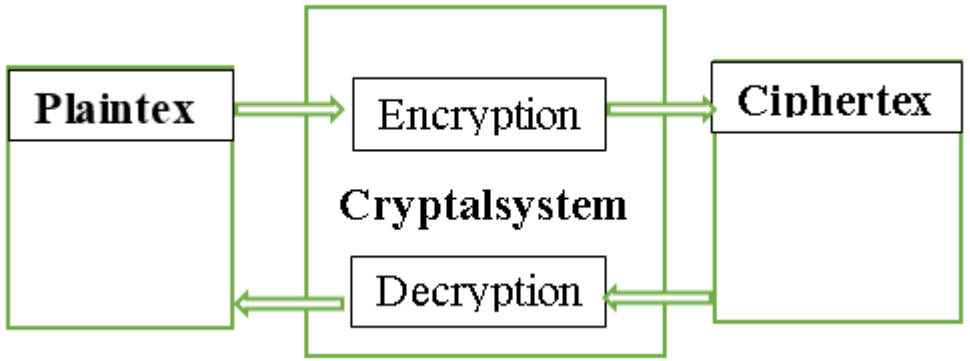
- **Decryption:** the process of recovering ciphertext into plaintext

- **Cipher:** is an algorithm to encrypt or decrypt information.

Basic Terminology

- **Cryptosystem:** The algorithms, or ciphers, used to encrypt and decrypt data

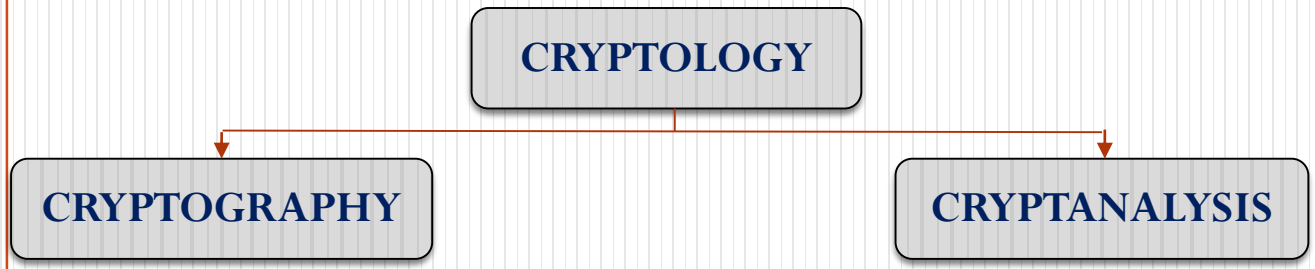
Cryptosystem = encryption + decryption algorithms.

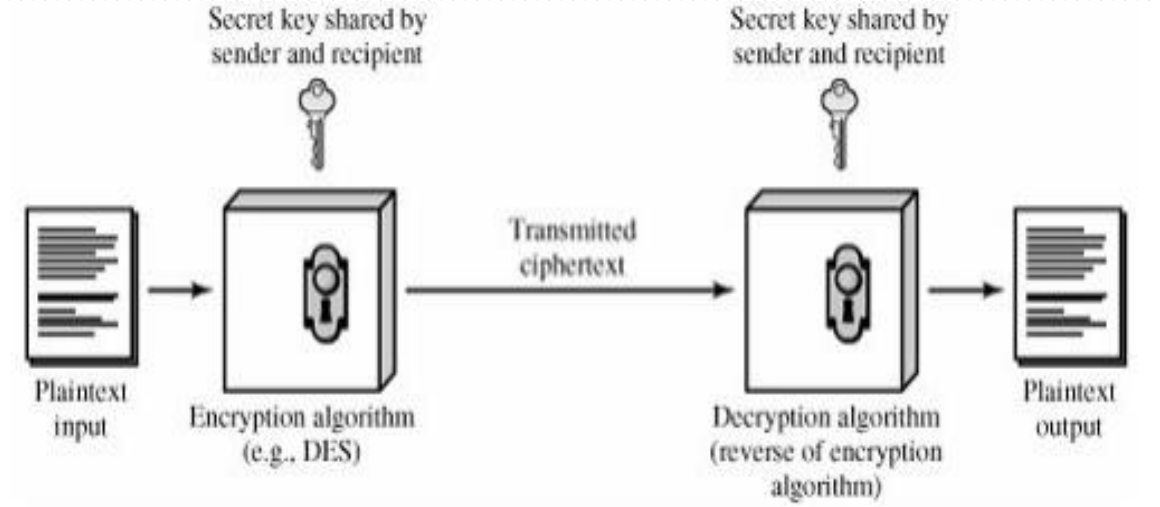


- **Key:** info used in cipher known only to sender/receiver

Cryptology

- **Cryptography:** is the Science of Writing in Secret Code
- **Cryptanalysis**: science of studying attacks against cryptographic systems
- **Cryptology = Cryptography + Cryptanalysis**





Cryptography

Cryptographic systems are characterized along three independent dimensions

- Type of encryption operations used
  - Substitution: replacing each element of the plaintext with another element
  - Transposition: rearranging the order of the elements of the plaintext.
  - Product: using multiple stages of substitutions and transpositions

Cryptography

- Number of keys used:
  - Symmetric: single-key, private-key
  - Asymmetric: two-key, public-key
- way in which the plaintext is processed
  - Block cipher: processes the input one block of elements at a time
  - Stream cipher: processes the input elements continuously

Cryptanalysis

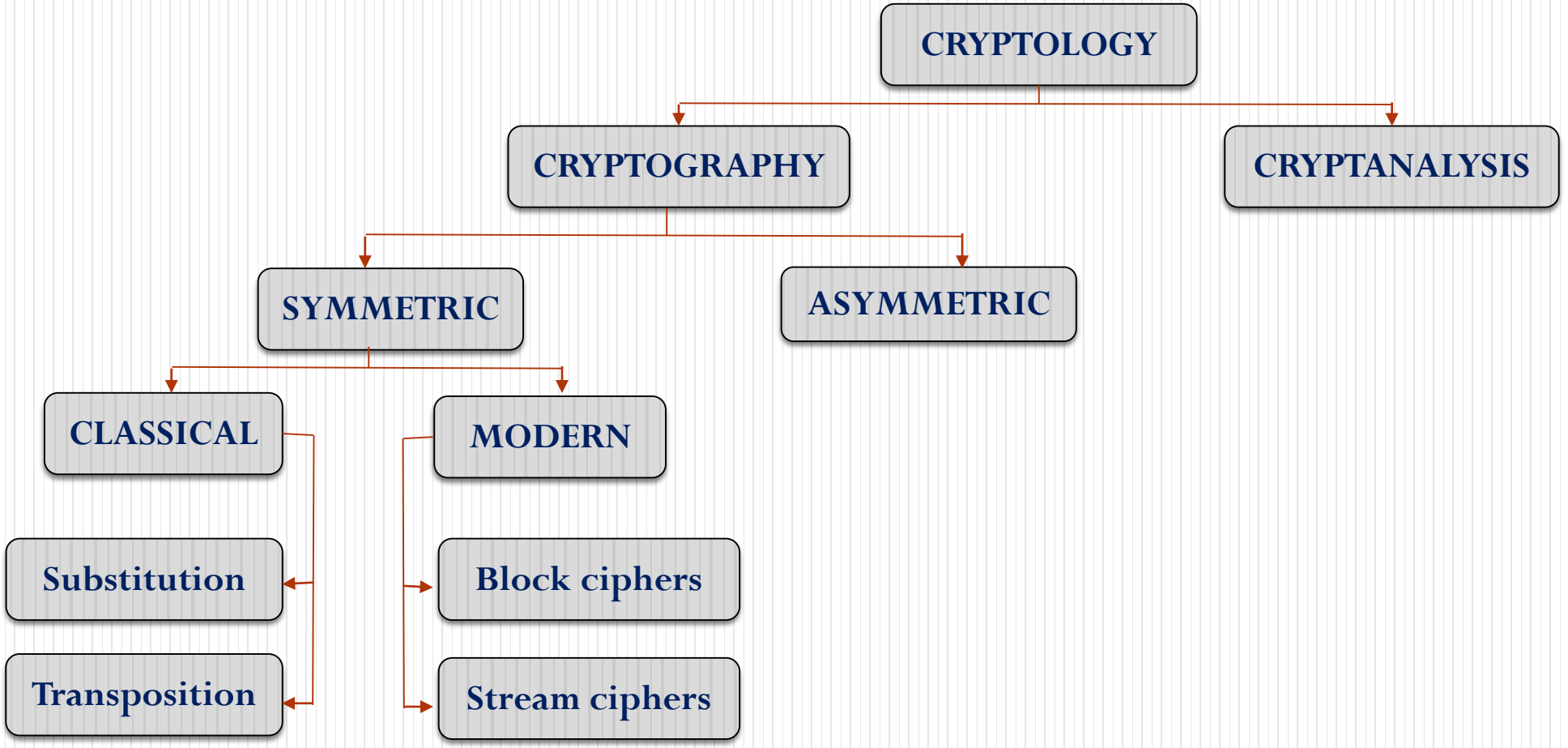
- **Objective**: to recover the key in use rather than simply to recover the plaintext of a single ciphertext.
- Two general approaches:
  - Brute-force attack: try every key to decipher the ciphertext
  - Cryptanalysis attack: exploit the characteristics of the algorithm to attempt to deduce a specific plaintext or to deduce the key being used

Cryptanalysis

- Brute-force attack: On average, half of all possible keys must be tried to achieve success.

Key Size (bits)	Number of Alternative Keys	Time Required at 1 Decryption/μs	Time Required at 10^6 Decryptions/μs
32	$2^{32} = 4.3 \times 10^9$	$2^{31}\mu s = 35.8$ minutes	2.15 milliseconds
56	$2^{56} = 7.2 \times 10^{16}$	$2^{55}\mu s = 1142$ years	10.01 hours
128	$2^{128} = 3.4 \times 10^{38}$	$2^{127}\mu s = 5.4 \times 10^{24}$ years	5.4×10^{18} years
168	$2^{168} = 3.7 \times 10^{50}$	$2^{167}\mu s = 5.9 \times 10^{36}$ years	5.9×10^{30} years
26 characters (permutation)	$26! = 4 \times 10^{26}$	$2 \times 10^{26}\mu s = 6.4 \times 10^{12}$ years	6.4×10^{6} years

Cryptology

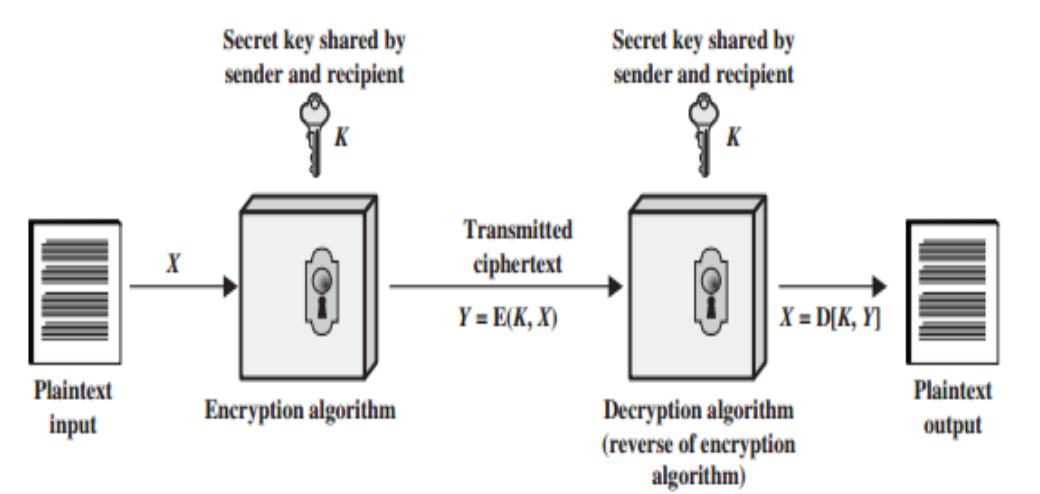

CRYPTOLOGY
CRYPTOGRAPHY
CRYPTANALYSIS
SYMMETRIC
ASYMMETRIC
CLASSICAL
MODERN
Substitution
Transposition
Block ciphers
Stream ciphers

2. Symmetric Ciphers

A. Classical Encryption
B. Modern Encryption

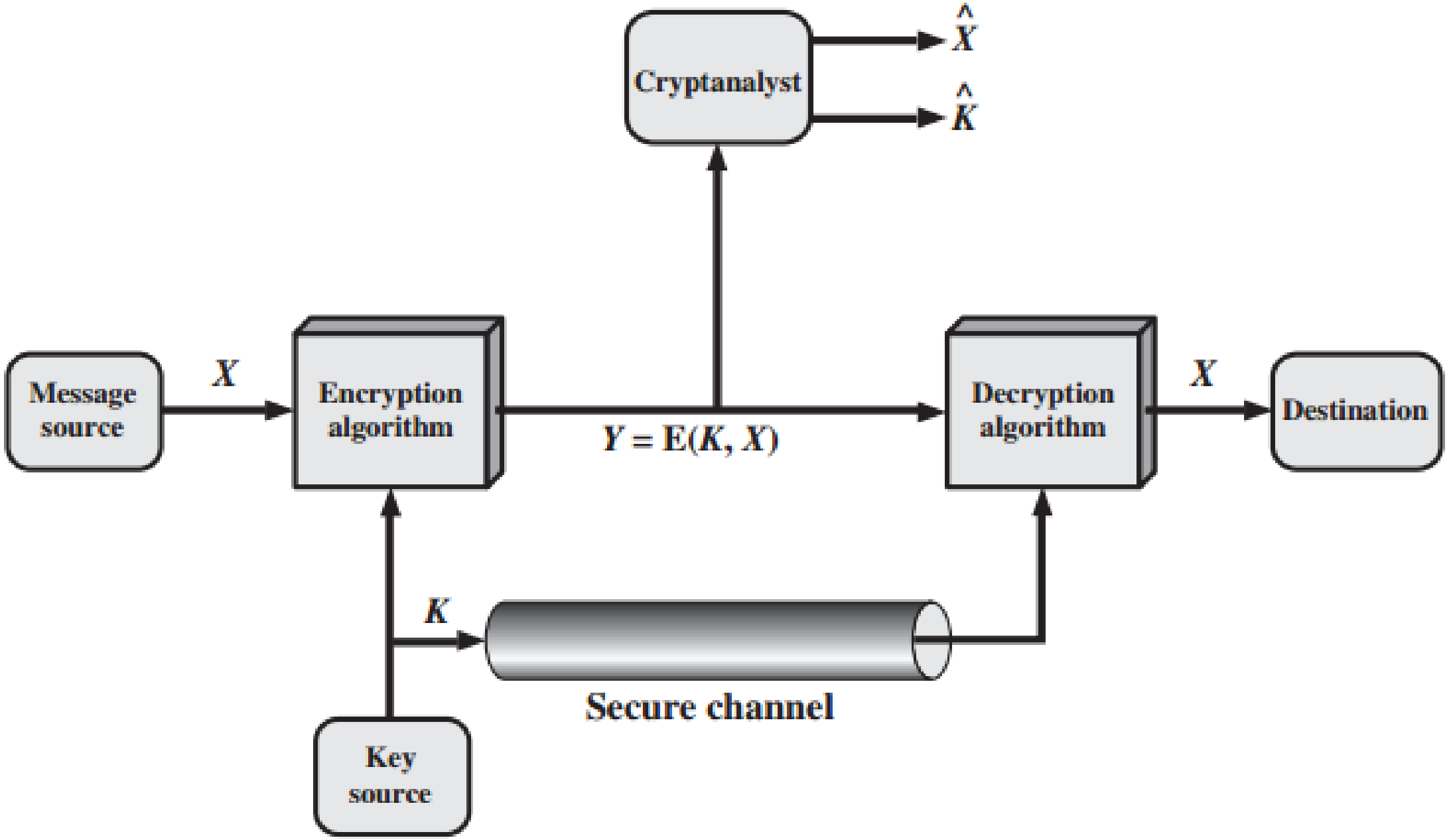
SYMMETRIC CIPHER MODEL

- Plaintext
- Encryption algorithm
- Secret key
- Ciphertext
- Decryption algorithm



Requirements

- Two requirements for secure use of symmetric encryption:
  - strong encryption algorithm
  - secret key know only to sender/receiver
- Mathematically have
  - Y=E(K,X)
  - X=D(K,Y)
- assume encryption algorithm is known
- implies a secure channel to distribute key


Cryptanalyst
$\hat{X}$
$\hat{K}$
Message source
X
Encryption algorithm
Y = E(K, X)
Decryption algorithm
X
Destination
K
Secure channel
Key source

A. Classical encryption

- Caesar Cipher
- Monoalphabetic Ciphers
- Playfair Cipher
- Hill Cipher
- Polyalphabetic Ciphers
- One-Time Pad

Substitution Techniques

A. Classical encryption (Cont.)

- Transposition Techniques
- Steganography

Substitution Techniques

a) Caesar Cipher

- Invented by Julius Caesar.
- Each letter is replaced by the letter K positions further down the alphabet.

Example:

- Plaintext: **meet me after the toga party**
- Key: k=3

```
plain:  a b c d e f g h i j k l m n o p q r s t u v w x y z
cipher: D E F G H I J K L M N O P Q R S T U V W X Y Z A B C
```

- Ciphertext: **PHHW PH DIWHU WKH WRJD SDUWB**

a) Caesar Cipher

- Mathematically, map letter to numbers:

a	b	c	d	e	f	g	h	i	j	k	l	m
0	1	2	3	4	5	6	7	8	9	10	11	12

n	o	p	q	r	s	t	u	v	w	x	y	z
13	14	15	16	17	18	19	20	21	22	23	24	25

- $C = E(P, k) = (P + k) \bmod 26$
- $P = D(C, k) = (C - k) \bmod 26$

C: Ciphertext letter

P: Plaintext letter

K: Key [1-25]

Plaintext = “Go to the canteen after class”, k =4

Ciphertext = ???

Cryptanalysis of Caesar Cipher

- Key space: {1, …, 25}
- Vulnerable to brute-force attacks
- Example: Break ciphertext “UNOU YZGZK’

KEY	PHHW PH DIWHU WKH WRJD SDUWB
1	oggv og chvgt vjg vqic rctva
2	nffu nf bgufs uif uphb qbsuz
3	**meet me after the toga party**
4	ldds ld zesdq sgd snfz ozqsx
5	kccr kc ydrcp rfc rmey nyprw
6	jbbq jb xcqbo qeb qldx mxoqv
7	iaap ia wbpan pda pkcw lwnpu
8	hzzo hz vaozm ocz ojbv kvmot
9	gyyn gy uznyl nby niau julns
10	fxxm fx tymxk max mhzt itkmr
11	ewwl ew sxlwj lzw lgys hsjlq
12	dvvk dv rwkvi kyv kfxr grikp
13	cuuj cu qvjuh jxu jewq fqhjo
14	btti bt puitg iwt idvp epgin
15	assh as othsf hvs hcuo dofhm
16	zrrg zr nsgre gur gbtn cnegl
17	yqqf yq mrfqd ftq fasm bmdfk
18	xppe xp lqepc esp ezrl alcej
19	wood wo kpdob dro dyqk zkbdi
20	vnnc vn jocna cqn cxpj yjach
21	ummb um inbmz bpm bwoi xizbg
22	tlla tl hmaly aol avnh whyaf
23	skkz sk glzkx znk zumg vgxze
24	rjjy rj fkyjw ymj ytlf ufwyd
25	qiix qi ejxiv xli xske tevxc

b) Monoalphabetic Cipher

- A **permutation** of a finite set of elements is an ordered sequence of all the elements of , with each element appearing exactly once.

For example, if S={a,b,c}, there are six permutations of S:

abc, acb, bac, bca, cab, cba

There are **n!** permutations of a set of **n** elements

- Substitution characters are a random permutation of the 26 letters of the alphabet

Monoalphabetic Cipher

Plain	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Cipher	A	M	I	K	O	P	Q	R	S	T	U	V	W	X	Y	Z	B	C	D	E	F	G	H	J	L	N

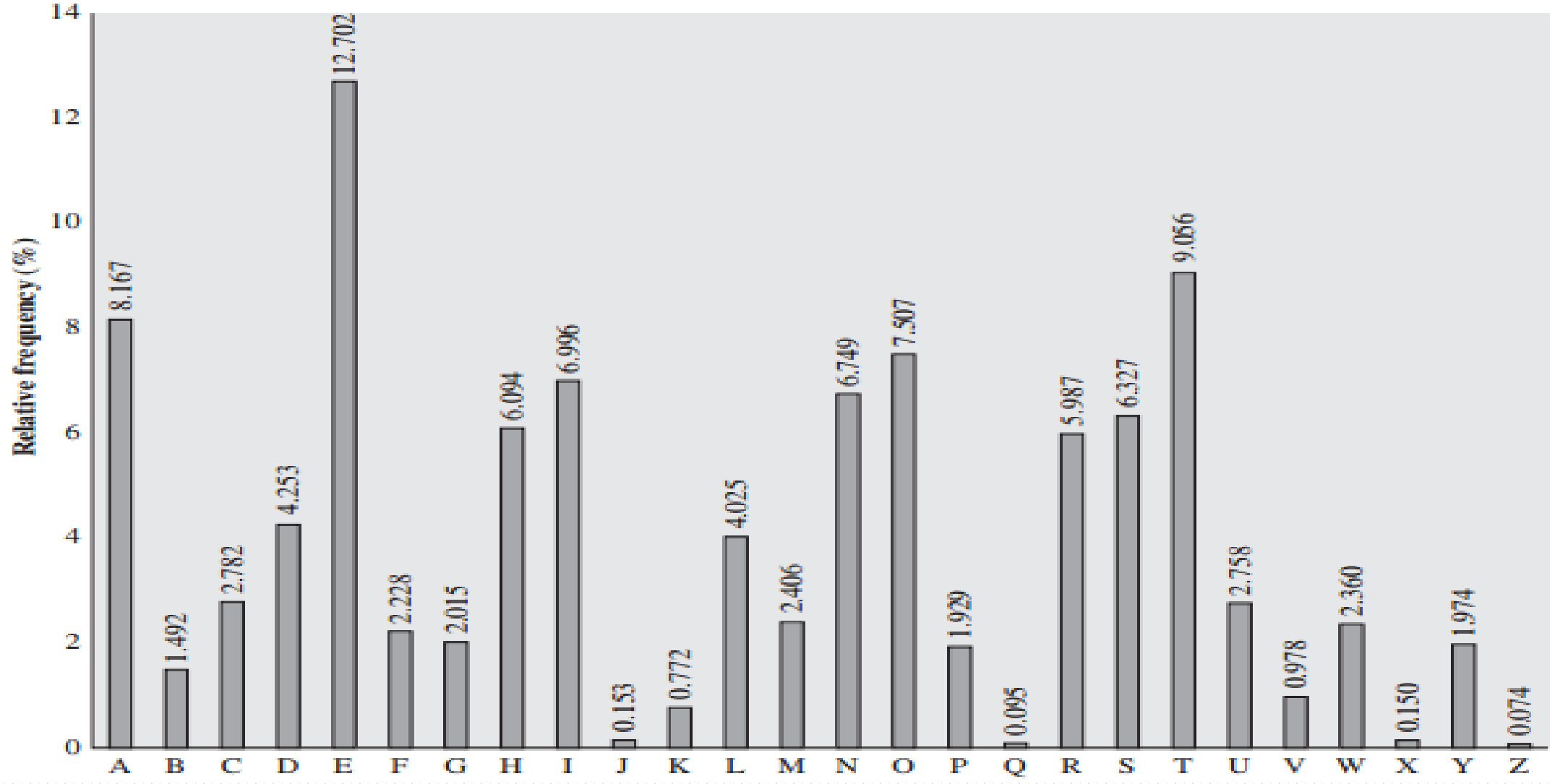
Plaintext: “**meet me after the toga party**”

Ciphertext: **???**

Monoalphabetic Cipher Security

- 26! Key
- It is secure against brute-force attacks.
- But not secure against some cryptanalytic attacks
- Problem is language characteristics.

Relative Frequency of Letters in English Text



Statistics for double & triple letters

Single Letter	Double Letter	Triple Letter
E	TH	THE
T	HE	AND
R	IN	TIO
N	ER	ATI
I	RE	FOR
O	ON	THA
A	AN	TER
S	EN	RES

Cryptanalysis of Monoalphabetic

- Example: The ciphertext to be solved is

UZQSOVUOHXMOPVGPOZPEVSGZWSZOPFPESXUDBMETSXAIZ

VUEPHZHMDZSHZOWSFPAPPDTSVPQUZWYMXUZUHSX

EPYEPOPDZSZUFPOMBZWPFUPZHMDJUDTMOHMQ

Cryptanalysis of Monoalphabetic

- The relative frequencies of the letters in the ciphertext (in percentages) are as follows:

P 13.33	H 5.83	F 3.33	B 1.67	C 0.00
Z 11.67	D 5.00	W 3.33	G 1.67	K 0.00
S 8.33	E 5.00	Q 2.50	Y 1.67	L 0.00
U 8.33	V 4.17	T 2.50	I 0.83	N 0.00
O 7.50	X 4.17	A 1.67	J 0.83	R 0.00
M 6.67				

- Guess {P, Z} – {E,T}
- {S, U, O, M, H} – {a, h, i, n, o, r, s}
- {A, B, G, Y, I, J }– {b, j, k, q, v, x, z}

Cryptanalysis of Monoalphabetic

```
UZQSOVUOHXMOPVGPOZPEVSGZWSZOPFPESXUDBMETSXAIZ
 t           e  e te  a t  t e e
VUEPHZHMDZSHZOWSFPAPPDTSVPQUZWYMXUZUHSX
   e t   t  t    e ee   e  t     t
EPYEPOPDZSZUFPOMBZWPFUPZHMDJUDTMOHMQ
 e  e e t t  e   t e   t
```

- Double letters: ZQ-1, ZP-1, ZW-3, ZO-2, ZH-2, ZS-2, ZU-2

=> Z->t; W->h

Cryptanalysis of Monoalphabetic

```
UZQSOVUOHXMOPVGPOZPEVSGZWSZOPFPESXUDBMETSXAIZ
 t a         e  e te  a that e e a         a
VUEPHZHMDZSHZOWSFPAPPDTSVPQUZWYMXUZUHSX
   e t   ta t ha e ee  a e  th    t  a
EPYEPOPDZSZUFPOMBZWPFUPZHMDJUDTMOHMQ
 e  e e tat  e   the   t
```

- Next, notice the sequence ZWSZ in the first line. th_t.

S->a

c) Playfair Cipher

- Invented by Charles Wheastone in 1854, but named after his friend Baron Playfair.
- Multiple-letter encryption cipher
- Security is much improved over the simple monoalphabetic cipher

Playfair Key Matrix

- Use a 5x5 matrix
- Fill in letter of the key (without duplicating letters)
- Fill the rest of matrix with other letters.

Example: key = MONARCHY

M	O	N	A	R
C	H	Y	B	D
E	F	G	I/J	K
L	P	Q	S	T
U	V	W	X	Z

Playfair Cipher

Plaintext is encrypted two letters at a time

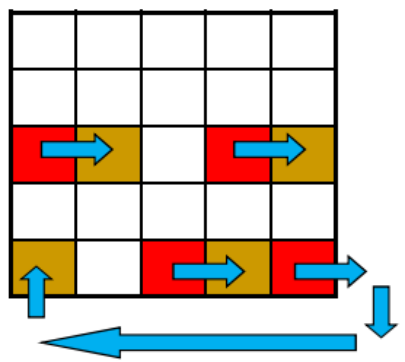
- Break the plaintext into pairs of two consecutive letters
- If a pair is a repeated letter, insert filler like ‘x’
- If both letters fall in the same row, replace each with the letter to its right (circularly)
- If both letters fall in the same column, replace each with the letter below it (circularly).
- Otherwise, each letter is placed by the letter in the same row but in the column of the other letter of the pair

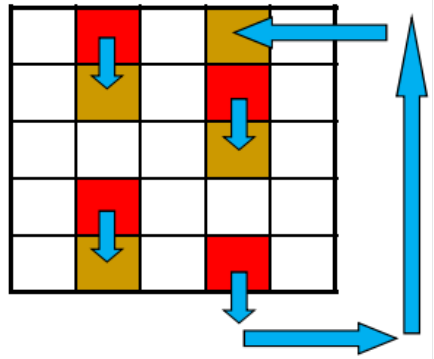
Playfair Cipher

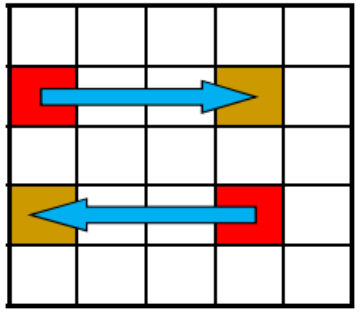
Example:

- Plaintext: **balloon**

=> ba lx lo on







Playfair Cipher

Example:

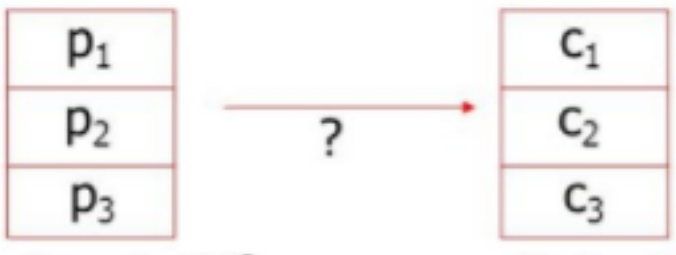
- Plaintext: **“hide the gold in the tree stump”**
- Key: **“playfair example”**
- Ciphertext **?**

Security of playfair cipher

- Security much improved over monoalphabetic
  - There are 26 x 26 = 676 diagrams
- Needs a 676 entry diagram frequency table to analyse and correspondingly more cipher text
- Widely used for many years
- Can be broken, give a few hundred letters
  - Still has much of plaintext structure

d) Hill Cipher

- Invented by Lester Hill in 1929
- Multi-letter copher



- Linear algebra: we are concerned with matrix arithmetic modulo 26

The inverse M^{-1} of a square matrix M(m x m):

$$MM^{-1} = M^{-1}M = I_m$$

Hill Cipher

$$\mathbf{A} = \begin{pmatrix} 5 & 8 \\ 17 & 3 \end{pmatrix} \qquad \mathbf{A}^{-1} \bmod 26 = \begin{pmatrix} 9 & 2 \\ 1 & 15 \end{pmatrix}$$

$$\mathbf{A}\mathbf{A}^{-1} = \begin{pmatrix} (5 \times 9) + (8 \times 1) & (5 \times 2) + (8 \times 15) \\ (17 \times 9) + (3 \times 1) & (17 \times 2) + (3 \times 15) \end{pmatrix}$$

$$= \begin{pmatrix} 53 & 130 \\ 156 & 79 \end{pmatrix} \bmod 26 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Hill Cipher

- The encryption algorithm takes m successive plaintext letters and substitutes for them m ciphertext letters.
- Each character is assigned a numerical value: (a =0, b =1, c, … z =25)

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

- Encryption: C = PK mod26
- Decryption: $P = K^{-1}C \mod 26$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \quad mod\ 26$$

Hill Cipher

- Example:

Plaintext: **paymoremoney**

Key:

$$K = \begin{bmatrix} 17 & 17 & 5 \\ 21 & 18 & 21 \\ 2 & 2 & 19 \end{bmatrix}$$

The first three letters of the plaintext are represented by the vector (15, 0, 24)

Hill Cipher

- **Encryption**: C = PK mod26

$$\begin{bmatrix} 17 & 17 & 5 \\ 21 & 18 & 21 \\ 2 & 2 & 19 \end{bmatrix} \begin{bmatrix} 15 \\ 0 \\ 24 \end{bmatrix} \; mod\, 26 = \begin{bmatrix} 11 \\ 13 \\ 18 \end{bmatrix} = \text{LNS}$$

- The ciphertext for the entire plaintext is **LNS???**

Hill Cipher

- **Decryption** $P = K^{-1}C \mod 26$

$$K^{-1} = \begin{bmatrix} 4 & 9 & 15 \\ 15 & 17 & 6 \\ 24 & 0 & 17 \end{bmatrix}$$

Vì :

$$\begin{bmatrix} 4 & 9 & 15 \\ 15 & 17 & 6 \\ 24 & 0 & 17 \end{bmatrix} \begin{bmatrix} 17 & 17 & 5 \\ 21 & 18 & 21 \\ 2 & 2 & 19 \end{bmatrix} = \begin{bmatrix} 443 & 442 & 442 \\ 858 & 495 & 780 \\ 494 & 52 & 365 \end{bmatrix} \; mod\ 26 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Khi đó bảng giải mã là: $K^{-1}C \mod 26 = K^{-1}KP \; mod\ 26 = P$

e) Polyalphabetic Ciphers

- Another way to improve on the simple monoalphabetic technique is to use different monoalphabetic substitutions as one proceeds through the plaintext message.
- The features in common:
  - A set of related monoalphabetic substitution rules is used
  - A key determines which particular rule is chosen for a given transformation

Vigenere Cipher

- Simplest polyalphabetic substitution cipher.
- Plaintext: $P = p_0, p_1, p_2, \ldots, p_{n-1}$
- Key consisting of the sequence of letters

$$K = k_0, k_1, k_2, \ldots, k_{m-1}$$

m<n

- ciphertext letters

$$C = C_0, C_1, C_2, \ldots, C_{n-1}$$

Vigenere Cipher

$$C = C_0, C_1, C_2, \ldots, C_{n-1} = \mathrm{E}(K, P) = \mathrm{E}[(k_0, k_1, k_2, \ldots, k_{m-1}), (p_0, p_1, p_2, \ldots, p_{n-1})]$$
$$= (p_0 + k_0) \bmod 26, (p_1 + k_1) \bmod 26, \ldots, (p_{m-1} + k_{m-1}) \bmod 26,$$
$$(p_m + k_0) \bmod 26, (p_{m+1} + k_1) \bmod 26, \ldots, (p_{2m-1} + k_{m-1}) \bmod 26, \ldots$$

- Encryption:

$$C_i = (p_i + k_{i \bmod m}) \bmod 26$$

- Decryption

$$p_i = (C_i - k_{i \bmod m}) \bmod 26$$

Vigenere Cipher- Example:

Plaintext: ***we are discovered save yourself***

Key***: deceptive***

plaintext: `wearediscoveredsaveyourself`

key: `DECEPTIVEDECEPTIVEDECEPTIVE`

ciphertext: `ZICVTWQNGRZGVTWAVZHCQYGLMGJ`

key	3	4	2	4	15	19	8	21	4	3	4	2	4	15
plaintext	22	4	0	17	4	3	8	18	2	14	21	4	17	4
ciphertext	25	8	2	21	19	22	16	13	6	17	25	6	21	19

key	19	8	21	4	3	4	2	4	15	19	8	21	4
plaintext	3	18	0	21	4	24	14	20	17	18	4	11	5
ciphertext	22	0	21	25	7	2	16	24	6	11	12	6	9

f) One-Time Pad

- Joseph Mauborgne
- using a random key that is as long as the message.
- The key is to be used to encrypt and decrypt a single message, and then is

Discarded.

Plaintext P:	wearediscoveredsaveyourself
Key K1 :	FHWYKLVMKVKXCVKDJSFSAPXZCVP
Ciphertext :	BLWPOODEMJFBTZNVJNJQOJORGGU

One-Time Pad

Bản mã C:	BLWPOODEMJFBTZNVJNJQOJORGGU
Khóa K_2:	IESRLKBWJFCIFZUCJLZXAXAAPSY
Bản giải mã:	theydecidedtoattacktomorrow
	(*they decided to attack tomorrow*)
Bản mã C:	BLWPOODEMJFBTZNVJNJQOJORGGU
Khóa K_3:	FHAHDDRAIQFIASJGJWQSVVBJAZB
Bản giải mã:	wewillmeetatthepartytonight
	(*we will meet at the party tonight*)

TRANSPOSITION TECHNIQUES

Transposition Techniques

- Performing some sort of permutation on the plaintext letters.
- The simplest such cipher is the **rail fence** technique: plaintext is written down as a sequence of diagonals and then read off as a sequence of rows.
- For Example: “meet me after the toga party”

M		E		M		A		T		R		H		T		G		P		R		Y
	E		T		E		F		E		T		A		O		A		A		T	

Encrypted message: MEMATRHTGPRYETEFETEOAAT

Transposition Techniques

- A more complex scheme is to write the message in a rectangle, row by row, and read the message off, column by column, but permute the order of the columns. The order of the columns then becomes the key to the algorithm

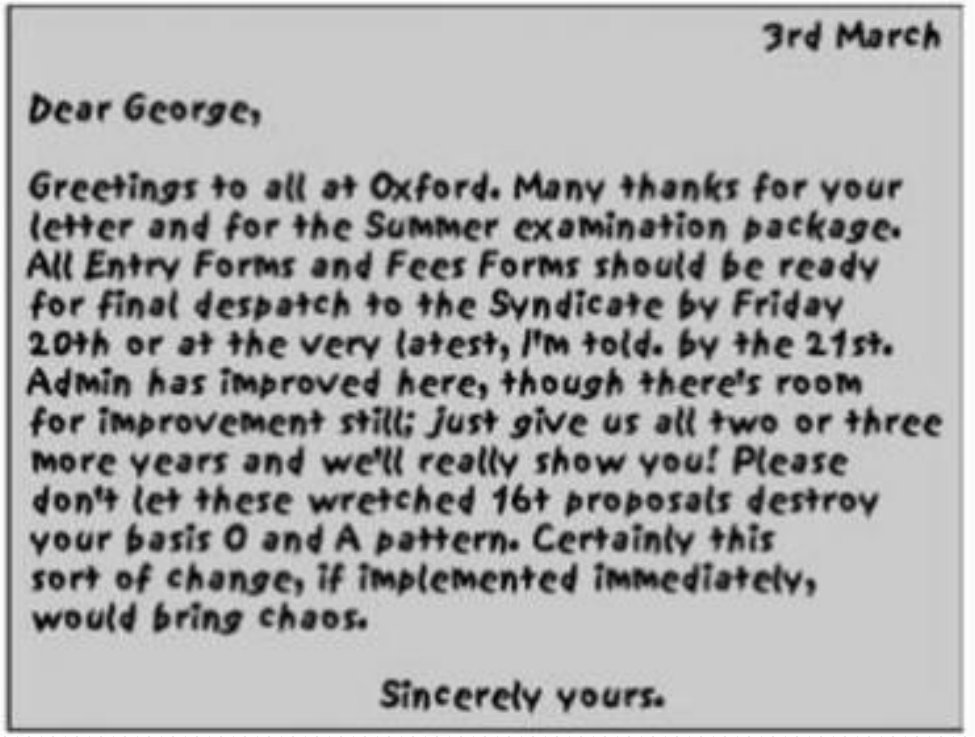
```
Key:          4 3 1 2 5 6 7
Plaintext:    a t t a c k p
              o s t p o n e
              d u n t i l t
              w o a m x y z
Ciphertext:   TTNAAPTMTSUOAODWCOIXKNLYPETZ
```

Steganography

Steganography

- Steganography is data hidden within data

Hidden message:
Your package ready Friday 21st room three. Please destroy this immediately



3rd March

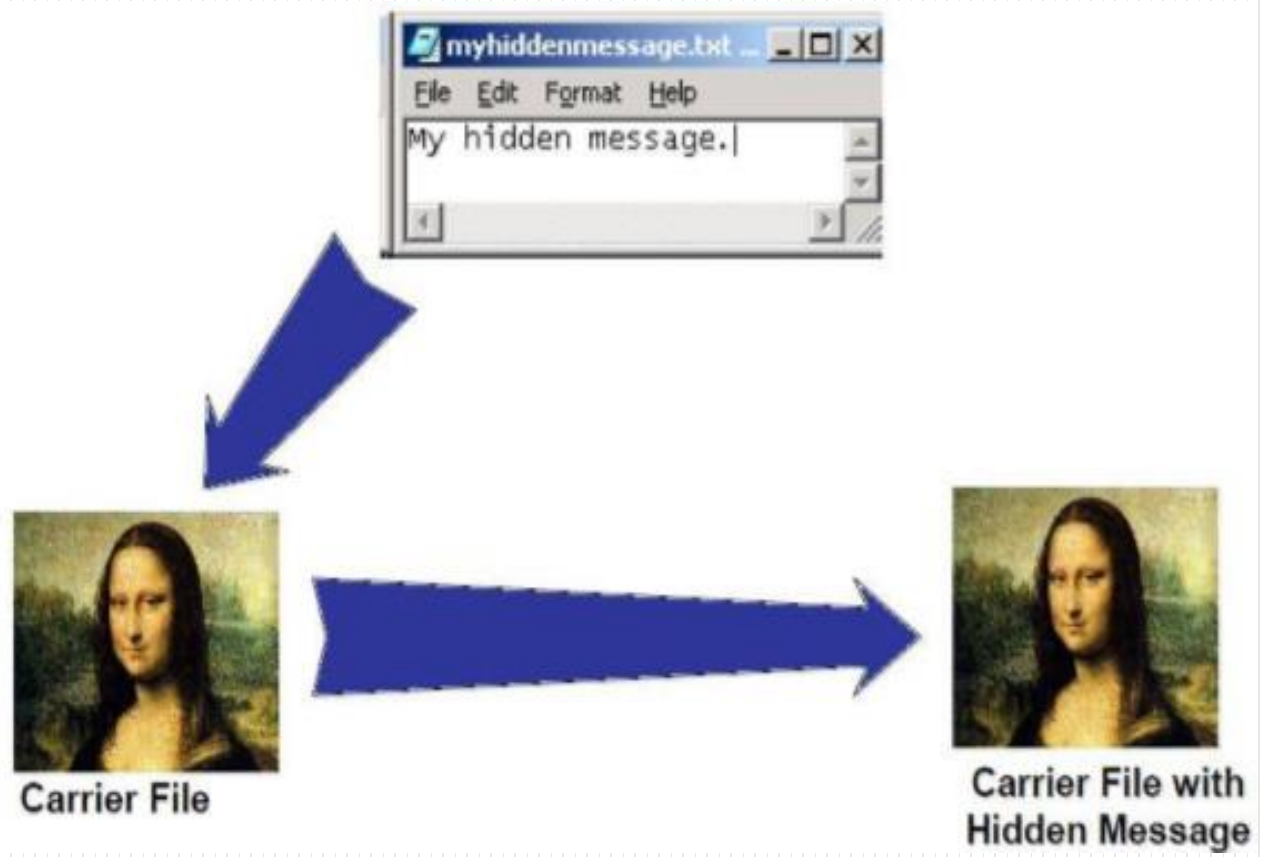
Dear George,

Greetings to all at Oxford. Many thanks for your letter and for the Summer examination package. All Entry Forms and Fees Forms should be ready for final despatch to the Syndicate by Friday 20th or at the very latest, I'm told. by the 21st. Admin has improved here, though there's room for improvement still; just give us all two or three more years and we'll really show you! Please don't let these wretched 16t proposals destroy your basis O and A pattern. Certainly this sort of change, if implemented immediately, would bring chaos.

Sincerely yours.

Steganography (cont.)

- **Tools**:
  - Image Steganograph
  - Xiao Steganography
  - Steghide
  - Crypture



Thanks